4.8 Perform Congruence Transformations

Before You determined whether two triangles are congruent.

Now You will create an image congruent to a given triangle.

Why So you can describe chess moves, as in Ex. 41.

Key Vocabulary
- **transformation**
- **image**
- **translation**
- **reflection**
- **rotation**
- **congruence transformation**

A **transformation** is an operation that moves or changes a geometric figure in some way to produce a new figure. The new figure is called the **image.** A transformation can be shown using an arrow.

The order of the vertices in the transformation statement tells you that P is the image of A, Q is the image of B, and R is the image of C.

$$\triangle ABC \rightarrow \triangle PQR$$

Original figure — Image

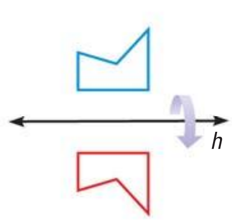
There are three main types of transformations. A **translation** moves every point of a figure the same distance in the same direction. A **reflection** uses a *line of reflection* to create a mirror image of the original figure. A **rotation** turns a figure about a fixed point, called the *center of rotation*.

EXAMPLE 1 Identify transformations

TRANSFORMATIONS You will learn more about transformations in Lesson 6.7 and in Chapter 9.

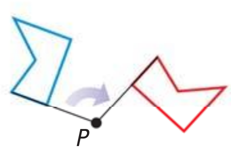
Name the type of transformation demonstrated in each picture.

a.



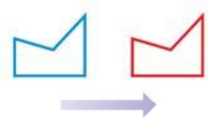
Reflection in a horizontal line

b.



Rotation about a point

c.



Translation in a straight path

✓ GUIDED PRACTICE for Example 1

1. Name the type of transformation shown.

CONGRUENCE Translations, reflections, and rotations are three types of *congruence transformations*. A **congruence transformation** changes the position of the figure without changing its size or shape.

TRANSLATIONS In a coordinate plane, a translation moves an object a given distance right or left and up or down. You can use coordinate notation to describe a translation.

READ DIAGRAMS
In this book, the original figure is blue and the transformation of the figure is red, unless otherwise stated.

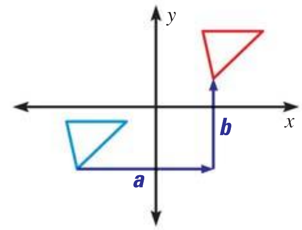
KEY CONCEPT — *For Your Notebook*

Coordinate Notation for a Translation

You can describe a translation by the notation

$(x, y) \rightarrow (x + a, y + b)$

which shows that each point (x, y) of the blue figure is translated horizontally a units and vertically b units.



EXAMPLE 2 Translate a figure in the coordinate plane

Figure *ABCD* has the vertices $A(-4, 3)$, $B(-2, 4)$, $C(-1, 1)$, and $D(-3, 1)$. Sketch *ABCD* and its image after the translation $(x, y) \rightarrow (x + 5, y - 2)$.

Solution

First draw *ABCD*. Find the translation of each vertex by adding 5 to its *x*-coordinate and subtracting 2 from its *y*-coordinate. Then draw *ABCD* and its image.

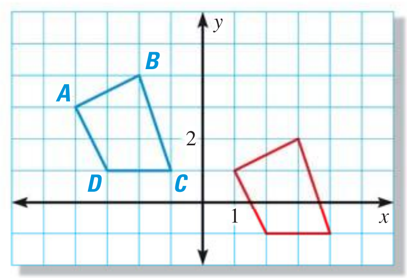
$(x, y) \rightarrow (x + 5, y - 2)$

$A(-4, 3) \rightarrow (1, 1)$

$B(-2, 4) \rightarrow (3, 2)$

$C(-1, 1) \rightarrow (4, -1)$

$D(-3, 1) \rightarrow (2, -1)$

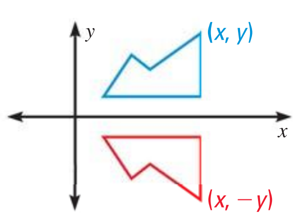


REFLECTIONS In this lesson, when a reflection is shown in a coordinate plane, the line of reflection is always the *x*-axis or the *y*-axis.

KEY CONCEPT — *For Your Notebook*

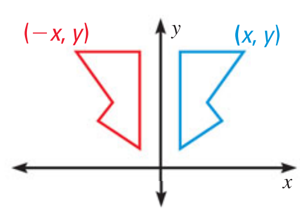
Coordinate Notation for a Reflection

Reflection in the *x*-axis



Multiply the *y*-coordinate by −1.
$(x, y) \rightarrow (x, -y)$

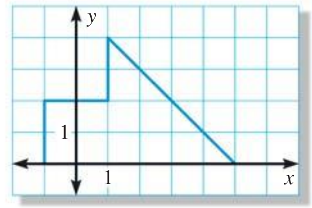
Reflection in the *y*-axis



Multiply the *x*-coordinate by −1.
$(x, y) \rightarrow (-x, y)$

EXAMPLE 3 Reflect a figure in the *y*-axis

WOODWORK You are drawing a pattern for a wooden sign. Use a reflection in the *x*-axis to draw the other half of the pattern.



Solution

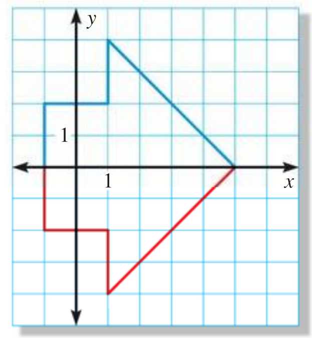
Multiply the *y*-coordinate of each vertex by −1 to find the corresponding vertex in the image.

$$(x, y) \rightarrow (x, -y)$$

$(-1, 0) \rightarrow (-1, 0)$ $\quad$ $(-1, 2) \rightarrow (-1, -2)$

$(1, 2) \rightarrow (1, -2)$ $\quad$ $(1, 4) \rightarrow (1, -4)$

$(5, 0) \rightarrow (5, 0)$



Use the vertices to draw the image. You can check your results by looking to see if each original point and its image are the same distance from the *x*-axis.

Animated Geometry at classzone.com

✓ GUIDED PRACTICE for Examples 2 and 3

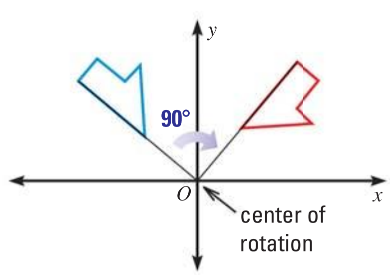
2. The vertices of $\triangle ABC$ are $A(1, 2)$, $B(0, 0)$, and $C(4, 0)$. A translation of $\triangle ABC$ results in the image $\triangle DEF$ with vertices $D(2, 1)$, $E(1, -1)$, and $F(5, -1)$. *Describe* the translation in words and in coordinate notation.

3. The endpoints of $\overline{RS}$ are $R(4, 5)$ and $S(1, -3)$. A reflection of $\overline{RS}$ results in the image $\overline{TU}$, with coordinates $T(4, -5)$ and $U(1, 3)$. Tell which axis $\overline{RS}$ was reflected in and write the coordinate rule for the reflection.

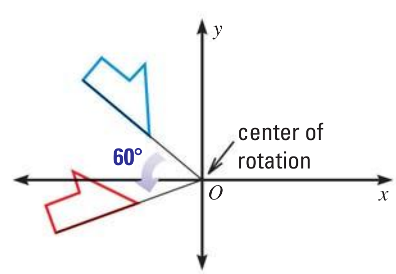
ROTATIONS In this lesson, if a rotation is shown in a coordinate plane, the center of rotation is the origin.

The direction of rotation can be either *clockwise* or *counterclockwise*. The *angle of rotation* is formed by rays drawn from the center of rotation through corresponding points on the original figure and its image.

90° clockwise rotation



60° counterclockwise rotation



Notice that rotations preserve distances from the center of rotation. So, segments drawn from the center of rotation to corresponding points on the figures are congruent.

EXAMPLE 4 Identify a rotation

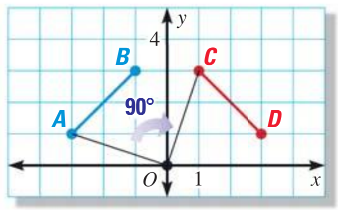
Graph $\overline{AB}$ and $\overline{CD}$. Tell whether $\overline{CD}$ is a rotation of $\overline{AB}$ about the origin. If so, give the angle and direction of rotation.

a. $A(-3, 1)$, $B(-1, 3)$, $C(1, 3)$, $D(3, 1)$

b. $A(0, 1)$, $B(1, 3)$, $C(-1, 1)$, $D(-3, 2)$

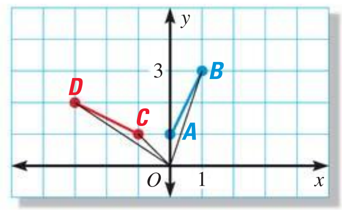
Solution

a.



$m\angle AOC = m\angle BOD = 90°$
This is a 90° clockwise rotation.

b.

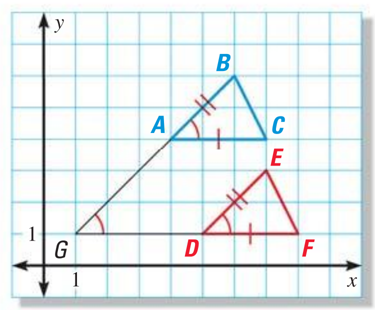


$m\angle AOC < m\angle BOD$
This is not a rotation.

EXAMPLE 5 Verify congruence

The vertices of $\triangle ABC$ are $A(4, 4)$, $B(6, 6)$, and $C(7, 4)$. The notation $(x, y) \rightarrow (x + 1, y - 3)$ describes the translation of $\triangle ABC$ to $\triangle DEF$. Show that $\triangle ABC \cong \triangle DEF$ to verify that the translation is a congruence transformation.

Solution



S You can see that $AC = DF = 3$, so $\overline{AC} \cong \overline{DF}$.

A Using the slopes, $\overline{AB} \parallel \overline{DE}$ and $\overline{AC} \parallel \overline{DF}$. If you extend $\overline{AB}$ and $\overline{DF}$ to form $\angle G$, the Corresponding Angles Postulate gives you $\angle BAC \cong \angle G$ and $\angle G \cong \angle EDF$. Then, $\angle BAC \cong \angle EDF$ by the Transitive Property of Congruence.

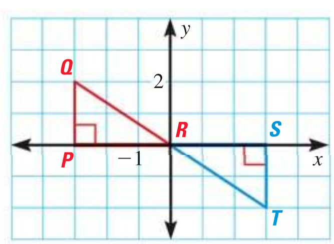
S Using the Distance Formula, $AB = DE = 2\sqrt{2}$ so $\overline{AB} \cong \overline{DE}$. So, $\triangle ABC \cong \triangle DEF$ by the SAS Congruence Postulate.

▶ Because $\triangle ABC \cong \triangle DEF$, the translation is a congruence transformation.

✓ GUIDED PRACTICE for Examples 4 and 5

4. Tell whether $\triangle PQR$ is a rotation of $\triangle STR$. If so, give the angle and direction of rotation.

5. Show that $\triangle PQR \cong \triangle STR$ to verify that the transformation is a congruence transformation.



4.8 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 23, and 39

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 25, 40, 41, and 43

SKILL PRACTICE

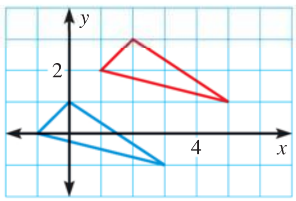
1. **VOCABULARY** *Describe* the translation $(x, y) \rightarrow (x - 1, y + 4)$ in words.

2. ★ **WRITING** *Explain* why the term *congruence transformation* is used in describing translations, reflections, and rotations.

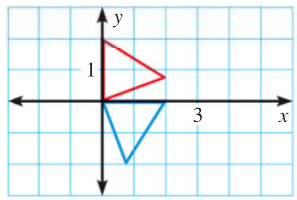
EXAMPLE 1 on p. 272 for Exs. 3–8

IDENTIFYING TRANSFORMATIONS **Name the type of transformation shown.**

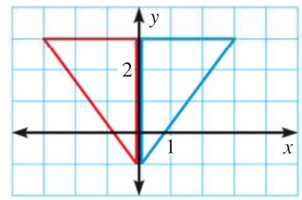
3.



4.



5.



WINDOWS **Decide whether the moving part of the window is a translation.**

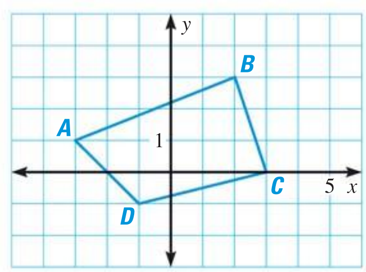
6. Double hung

7. Casement

8. Sliding

EXAMPLE 2 on p. 273 for Exs. 9–16

DRAWING A TRANSLATION **Copy figure *ABCD* and draw its image after the translation.**



9. $(x, y) \rightarrow (x + 2, y - 3)$

10. $(x, y) \rightarrow (x - 1, y - 5)$

11. $(x, y) \rightarrow (x + 4, y + 1)$

12. $(x, y) \rightarrow (x - 2, y + 3)$

COORDINATE NOTATION **Use coordinate notation to *describe* the translation.**

13. 4 units to the left, 2 units down

14. 6 units to the right, 3 units up

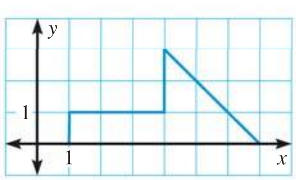
15. 2 units to the right, 1 unit down

16. 7 units to the left, 9 units up

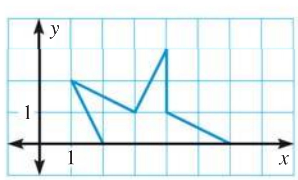
EXAMPLE 3 on p. 274 for Exs. 17–19

DRAWING **Use a reflection in the *x*-axis to draw the other half of the figure.**

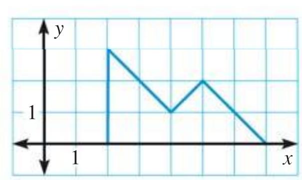
17.



18.



19.



EXAMPLE 4
on p. 275
for Exs. 20–23

ROTATIONS **Use the coordinates to graph $\overline{AB}$ and $\overline{CD}$. Tell whether $\overline{CD}$ is a rotation of $\overline{AB}$ about the origin. If so, give the angle and direction of rotation.**

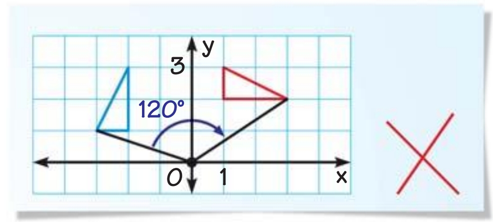
20. $A(1, 2), B(3, 4), C(2, -1), D(4, -3)$

21. $A(-2, -4), B(-1, -2), C(4, 3), D(2, 1)$

22. $A(-4, 0), B(4, -4), C(4, 4), D(0, 4)$

23. $A(1, 2), B(3, 0), C(2, -1), D(2, -3)$

24. ERROR ANALYSIS A student says that the red triangle is a 120° clockwise rotation of the blue triangle about the origin. *Describe* and correct the error.



25. ★ WRITING Can a point or a line segment be its own image under a transformation? *Explain* and illustrate your answer.

APPLYING TRANSLATIONS **Complete the statement using the description of the translation. In the description, points (0, 3) and (2, 5) are two vertices of a hexagon.**

26. If (0, 3) translates to (0, 0), then (2, 5) translates to __?__.

27. If (0, 3) translates to (1, 2), then (2, 5) translates to __?__.

28. If (0, 3) translates to (−3, −2), then (2, 5) translates to __?__.

xy **ALGEBRA** **A point on an image and the translation are given. Find the corresponding point on the original figure.**

29. Point on image: (4, 0); translation: $(x, y) \rightarrow (x + 2, y - 3)$

30. Point on image: (−3, 5); translation: $(x, y) \rightarrow (-x, y)$

31. Point on image: (6, −9); translation: $(x, y) \rightarrow (x - 7, y - 4)$

32. CONGRUENCE Show that the transformation in Exercise 3 is a congruence transformation.

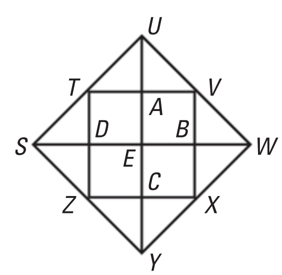
DESCRIBING AN IMAGE **State the segment or triangle that represents the image. You can use tracing paper to help you see the rotation.**

33. 90° clockwise rotation of $\overline{ST}$ about E

34. 90° counterclockwise rotation of $\overline{BX}$ about E

35. 180° rotation of $\triangle BWX$ about E

36. 180° rotation of $\triangle TUA$ about E



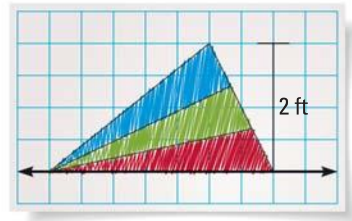
37. CHALLENGE Solve for the variables in the transformation of $\overline{AB}$ to $\overline{CD}$ and then to $\overline{EF}$.

$A(2, 3)$, $B(4, 2a)$ → Translation: $(x, y) \rightarrow (x - 2, y + 1)$ → $C(m - 3, 4)$, $D(n - 9, 5)$ → Reflection: in x-axis → $E(0, g - 6)$, $F(8h, -5)$

PROBLEM SOLVING

EXAMPLE 3
on p. 274
for Ex. 38

38. KITES The design for a kite shows the layout and dimensions for only half of the kite.



a. What type of transformation can a designer use to create plans for the entire kite?

b. What is the maximum width of the entire kite?

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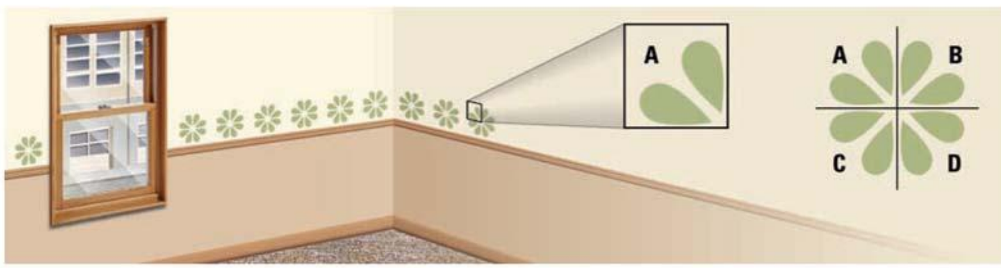
39. STENCILING You are stenciling a room in your home. You want to use the stencil pattern below on the left to create the design shown. Give the angles and directions of rotation you will use to move the stencil from *A* to *B* and from *A* to *C*.



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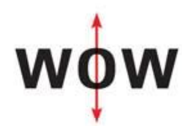
40. ★ **OPEN-ENDED MATH** Some words reflect onto themselves through a vertical line of reflection. An example is shown.

a. Find two other words with vertical lines of reflection. Draw the line of reflection for each word.



b. Find two words with horizontal lines of reflection. Draw the line of reflection for each word.

41. ★ **SHORT RESPONSE** In chess, six different kinds of pieces are moved according to individual rules. The Knight (shaped like a horse) moves in an "L" shape. It moves two squares horizontally or vertically and then one additional square perpendicular to its original direction. When a knight lands on a square with another piece, it *captures* that piece.

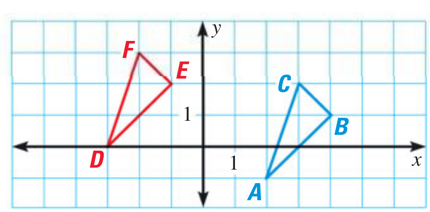
a. *Describe* the translation used by the Black Knight to capture the White Pawn.

b. *Describe* the translation used by the White Knight to capture the Black Pawn.

c. After both pawns are captured, can the Black Knight capture the White Knight? *Explain*.

EXAMPLE 5
on p. 275
for Ex. 42

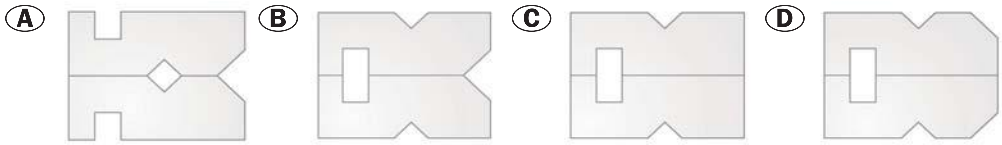
42. VERIFYING CONGRUENCE Show that $\triangle ABC$ and $\triangle DEF$ are right triangles and use the HL Congruence Theorem to verify that $\triangle DEF$ is a congruence transformation of $\triangle ABC$.



○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

43. ★ **MULTIPLE CHOICE** A piece of paper is folded in half and some cuts are made, as shown. Which figure represents the unfolded piece of paper?



44. **CHALLENGE** A triangle is rotated 90° counterclockwise and then translated three units up. The vertices of the final image are $A(-4, 4)$, $B(-1, 6)$, and $C(-1, 4)$. Find the vertices of the original triangle. Would the final image be the same if the original triangle was translated 3 units up and then rotated 90° counterclockwise? *Explain* your reasoning.

MIXED REVIEW

PREVIEW
Prepare for Lesson 5.1 in Exs. 45–50.

Simplify the expression. Variables *a* and *b* are positive.

45. $\frac{-a - 0}{0 - (-b)}$ *(p. 870)* **46.** $|(a + b) - a|$ *(p. 870)* **47.** $\frac{2a + 2b}{2}$ *(p. 139)*

Simplify the expression. Variables *a* and *b* are positive. *(p. 139)*

48. $\sqrt{(-b)^2}$ **49.** $\sqrt{(2a)^2}$ **50.** $\sqrt{(2a - a)^2 + (0 - b)^2}$

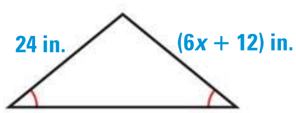
51. Use the SSS Congruence Postulate to show $\triangle RST \cong \triangle UVW$. *(p. 234)*

$R(1, -4)$, $S(1, -1)$, $T(6, -1)$ $U(1, 4)$, $V(1, 1)$, $W(6, 1)$

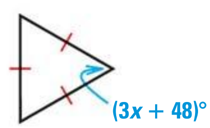
QUIZ for Lessons 4.7–4.8

Find the value of *x*. *(p. 264)*

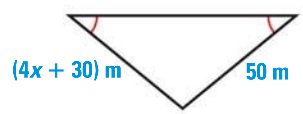
1.



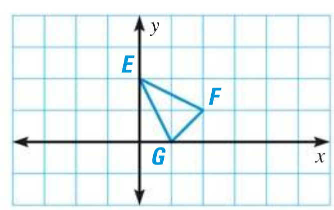
2.



3.



Copy $\triangle EFG$ and draw its image after the transformation. Identify the type of transformation. *(p. 272)*



4. $(x, y) \to (x + 4, y - 1)$ **5.** $(x, y) \to (-x, y)$

6. $(x, y) \to (x, -y)$ **7.** $(x, y) \to (x - 3, y + 2)$

8. Is Figure B a rotation of Figure A about the origin? If so, give the angle and direction of rotation. *(p. 272)*